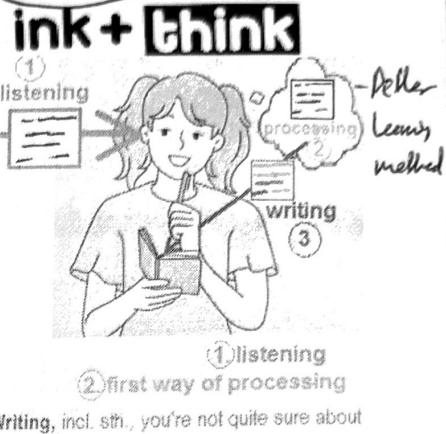
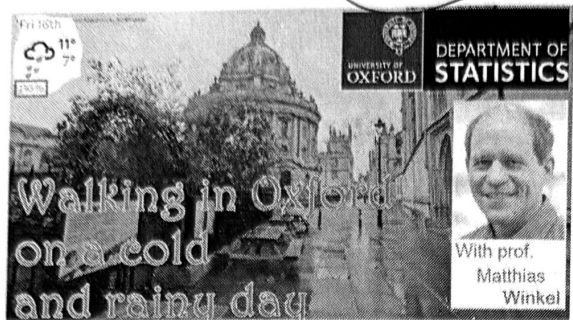


+0.1 +0.1

+0.1 11.2.26  
+0.1 5.3.26

+0.1 16.2.2026  
+0.1 17.2.2026



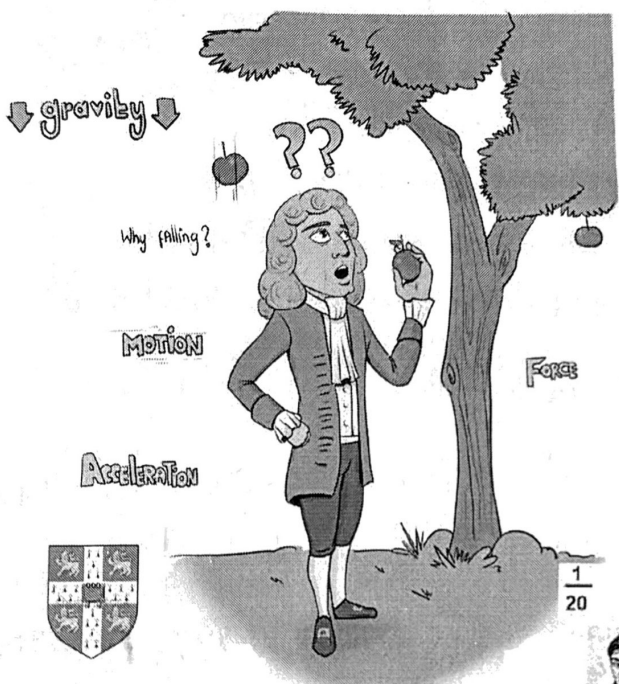
Focused on observable, physical phenomena.

School  $\downarrow$  gravity  $\downarrow$  MOTION

==formalism==> University  $E=MC^2$   $\int \vec{J} \cdot d\vec{s}$

# CONCRETE AND ABSTRACT THINKING

focus on pure mathematical equations. Instead of directly relying on direct sensory observations, he worked with abstract symbols, equations,



ISAAC NEWTON

$$W = 2\pi f$$

$$\beta = \frac{\Delta I_c}{\Delta I_a}$$

$$E = \frac{1}{2} h \nu / m$$

$$z = \frac{h}{h \nu}$$

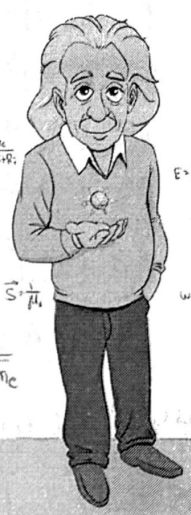
$$(a+n)^2$$

$$\oint \vec{D} \cdot d\vec{s} = q$$

$$\vec{s} = \frac{1}{\mu_0}$$

$$\lambda = \frac{h}{\sqrt{2eUm}}$$

$$E = \frac{1}{2} h \nu / m$$



ALBERT EINSTEIN



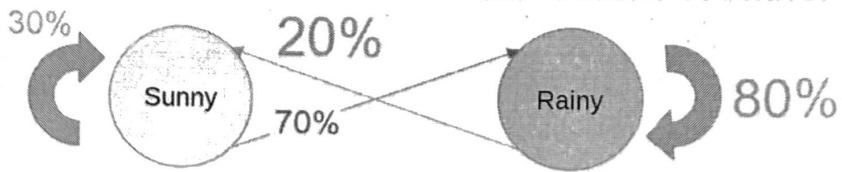
Motivation: 80% chance of rain  
Let  $A_j$  be the event of rain at 9am on day  $j$  of this term,  $1 \leq j \leq n$

Suppose the events  $A_j$  each have probability  $p$  independently

Oxford			
Tue 13th	Wed 14th	Thu 15th	Fri 16th
10° 9°	13° 10°	13° 8°	11° 7°
70%	70%	70%	80%

## Markoff Chain Probability Model

for Oxford Weather



If it rains today, there is an 80% chance it will be raining,

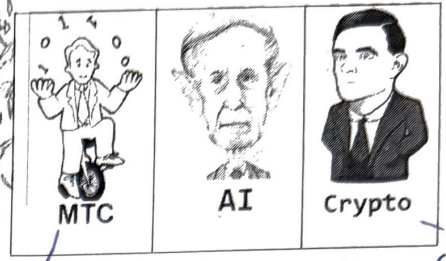
+0.1 7.9 +0.1 +0.1 +0.1 +0.1 +0.1 (circled) 7.9

(+0.1) Thorn

Do tip if I make error  
 Willingness to be corrected

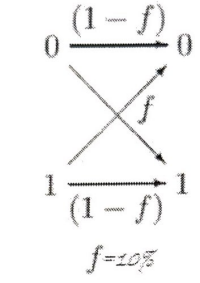


Sir Dr. D. MacKay,  
 University of Cambridge  
 (22 April 1967 – 14 April 2016)



"I believe in clean energy, but I also believe in mathematics"

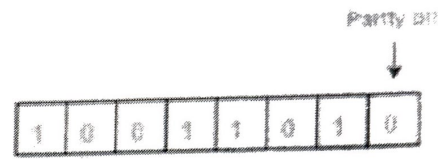
Redundant data helps ensure accuracy and reliability in the presence of errors.



Machine translation and coding parity bits for error detection



Transmitted data unit

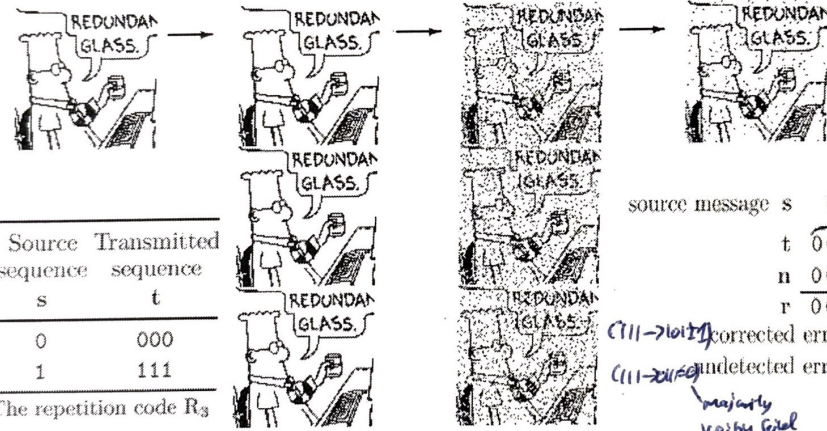


Transmitted data unit

(circled) 0.1

Whenever each bit is repeated.

S ENCODER t CHANNEL f = 10% r DECODER S



Repeat each bit multiple times to create redundancy.

Source sequence s	Transmitted sequence t
0	000
1	111

The repetition code R<sub>3</sub>

source message s	0	0	1	0	1	1	0
t	000	000	111	000	111	111	000
n	000	001	000	000	101	000	000
r	000	001	111	000	010	111	000

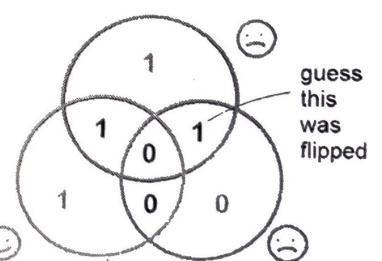
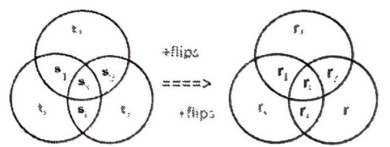
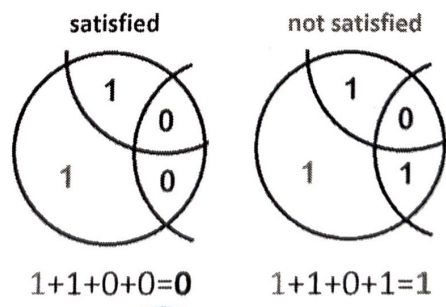
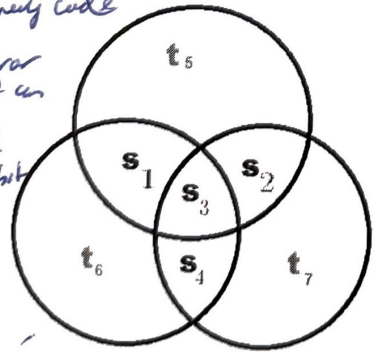
corrected errors \*  
 undetected errors \*  
 error \* -2 error

### 7.4. Hamming code.

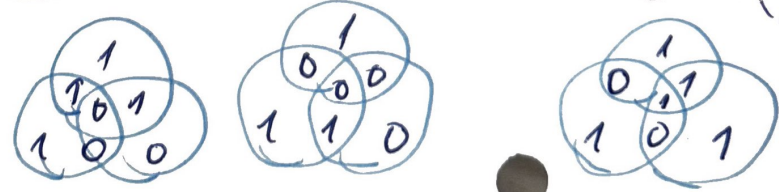
$$\frac{4}{\Sigma} \rightarrow \frac{7}{t}$$

type of binary code

used for error correction that can detect and correct single-bit errors.



Represents different sets of parity bits. The intersections of the data bits being protected by the parity bits.



- Flips 000

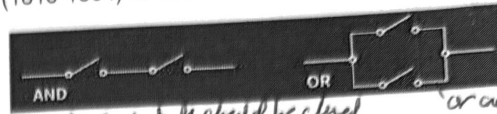
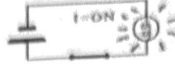
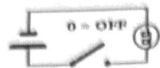


Massachusetts Institute of Technology (MIT)



Doc Lan Algebra

Lecture by Pr. Bob Gallager  
Boole (1815-1864) & Shannon (1916-2001)



AND - both should be closed

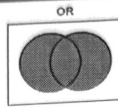
OR - one should be closed

Logical addition (disjunction)

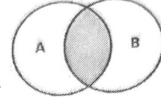
A	B	F=A∨B
0	0	0
0	1	1
1	0	1
1	1	1

A	B	A ∨ B
True	True	True
True	False	True
False	True	True
False	False	False

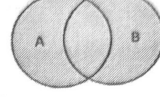
Logical disjunction



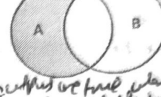
AND Both terms



OR Either term



NOT Only one term

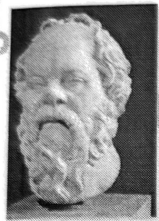


A∨B is true if A or B or both are true

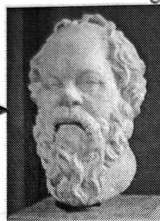
OR operation includes all elements that are in either set or both

either or true, one at least or at the very least

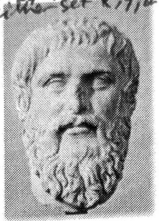
Good logic



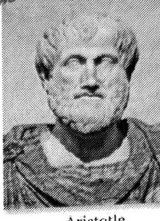
Socrates



Socrates



Plato



Aristotle



Socrates was a philosopher

philosophers are men

Socrates was a man

$S \in \Phi$

Socrates belongs to the set of philosophers ( $\Phi$ )

$\Phi \in A$

philosophers are a subset of men ( $A$ )

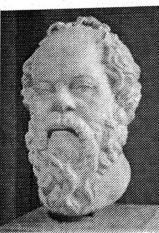
$S \in A$

Socrates belongs to the set of men

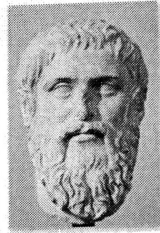
Bad logic



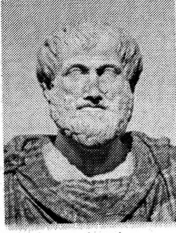
Socrates was a man



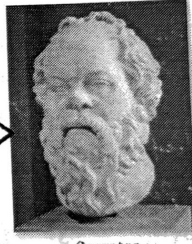
Socrates



Plato



Aristotle



Socrates

philosophers are men

Socrates was a philosopher

$S \in A$

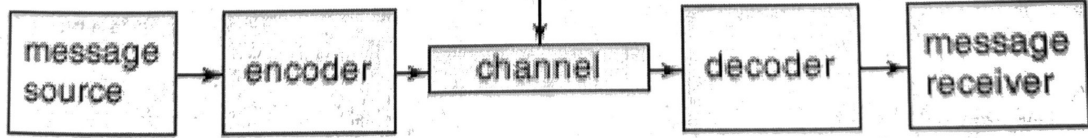
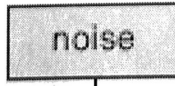
Socrates belongs to the set of men ( $A$ )

$\Phi \in A$

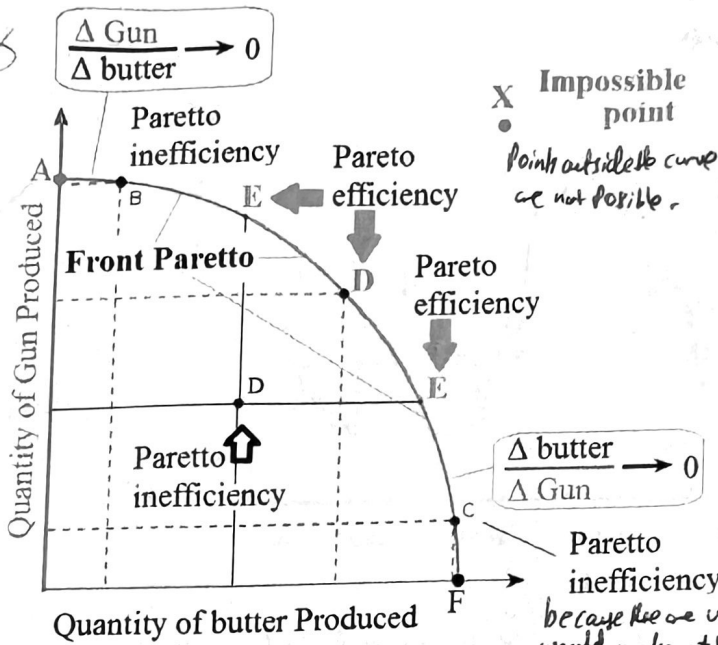
philosophers are a subset of men ( $A$ )

$S \in \Phi$

Socrates belongs to philosophers



Trade off between two goods guns and butter. Points on the curve show efficient allocations of resources.



A state where resources are allocated in the most efficient manner, such that any change to make one individual better off would make another worse off.



by Vilfredo Pareto  
1848-1923

The orange sector E-D-E is the most Pareto efficient - since an increase in one indicator leads to a decrease in another.

because there are unutilized resources and moving to a point on the curve would make at least one person better without making anyone worse off.

Why two rational individuals might not cooperate even if it appears to be in their best interests  
Prisoners' dilemma

		prisoner B	
		confess	remain silent
prisoner A	confess	5 years 5 years	0 year 20 years
	remain silent	20 years 0 year	1 year 1 year

confess, get 5 yrs, and he gets 0 and she 20 yrs, confess, last

Game Theory

Nash Equilibrium

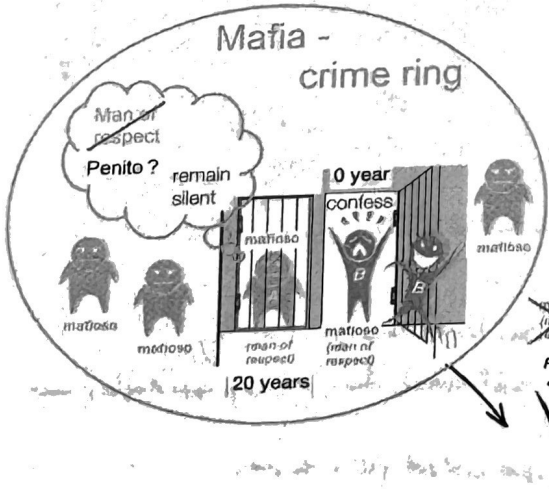


\*\* => Nash equilibrium

	H <sub>2</sub> (x)	Recognition;	Non-recognition;	
	Player 2			
H <sub>1</sub> (x)	Player 1	1	2	
Recognition;	1	-5*	0	
Non-recognition;	2	-20	-1	

if both self recognized, payoff is 5 each, if one remains silent and other confess, pay offs vary, if neither gets recognized, the payoff is -1 each.

-1-1  
Pareto Optimality

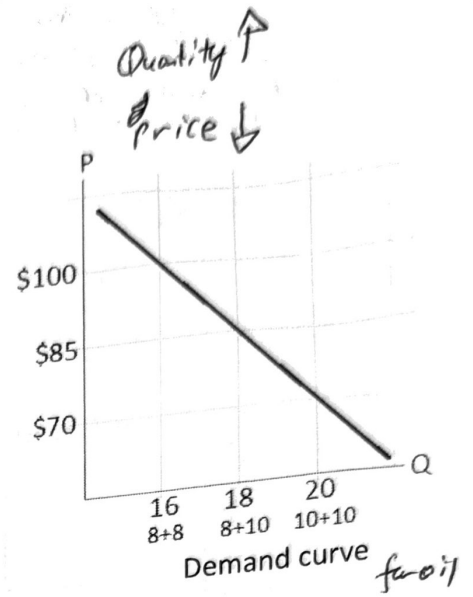


Nash equilibrium - a concept where no player can benefit by changing their strategy while the other players keep theirs unchanged.





















Significant fluctuation of oil prices,  
Peak 2008.

### Oil price hits 18-year low

Brent crude, US dollars per barrel



Payoff Matrix used to analyze the strategies and potential outcomes  
for two oil-producing entities

Barrel 		8 mill		10 mill	
		1. 	2. 	1. 	2. 
1. 	8 · 10 <sup>6</sup> 	 \$800 millions per day  \$100  \$800 millions per day	 \$850 millions per day  \$85  \$680		
	10 · 10 <sup>6</sup> 	 \$680 millions per day  \$85  \$850 millions per day	 \$700 millions per day  \$70  \$700 millions per day		

2 players & strategies.

John Nash works in game theory / won Nobel Peace Prize

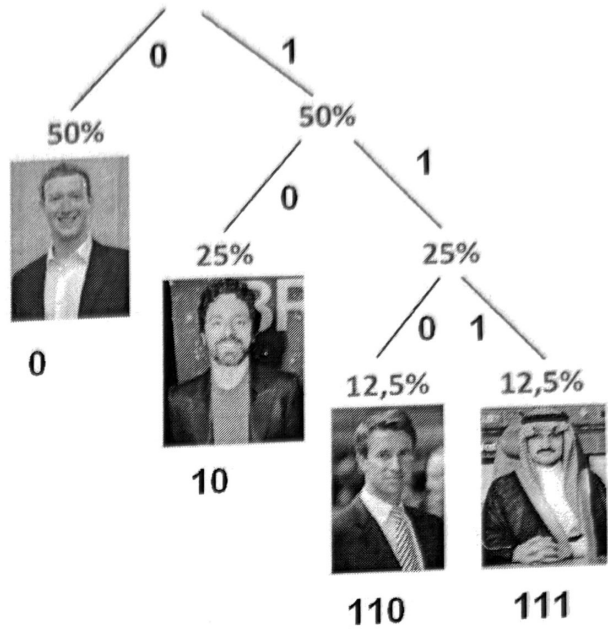


#### Optimal Strategies:

- If both players cooperate and limit their production to 8m. Joint revenue 1600m per day.
- If one player increases production to 10, other stays 8, total revenue decreases, but player producing more oil gains more market share.
- If both increase to 10 million, prices will drop, revenue will be 1400 per day.

# Huffman tree:

Tree shows the process of creating a Huffman coding tree. Each character is assigned a binary code based on its frequency. Characters with higher frequency are closer to the root, resulting in shorter codes. The binary codes are created by traversing the tree: left branches are 0 and right branches are 1.



This assumes each symbol is independent of each other.

First-order approximation (symbols independent but with frequencies of Belarusian text).

Мама мыла ра		
М - 3	30%	1-3 М
а - 4	40%	4-7 а
ы - 1	10%	8 -ы
л - 1	10%	9 -л
р - 1	10%	10 -р
10		
ла	ма	ма
ма	ма	ра

Мама мыла ра		
Ма - 2	22%	1-2 ма
ам - 2	22%	3-4 ам
мы - 1	11%	5 мы
ыл - 1	11%	6 ыл
ла - 1	11%	7 ла
ар - 1	11%	8 ар
ра - 1	11%	9 ра
9		

Considers pairs of symbols where the probability of a symbol depends on the previous symbol.

break down of individual symbols into frequencies.

number of pairs starting in the text

0	4	6	7	3	1	9	1	6	7	3	5
ам	ыл	ла	ам	ма	ра	ма	ыл	ла	ам	мы	
мы	ла	ра									



Order approximation (digram (2-symbols) structure as in Belarusian)

+0.1 +0.1



Say NO to the first

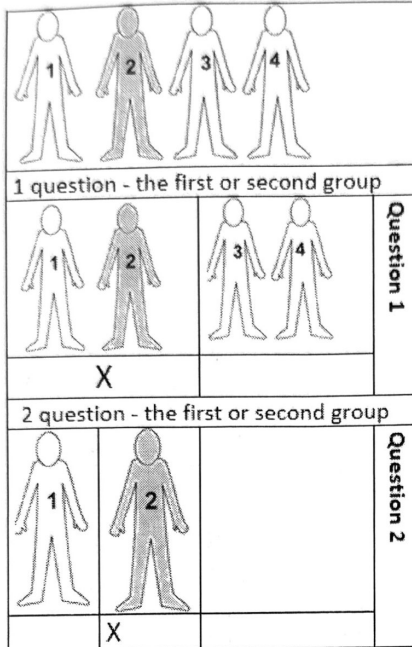


Say YES to the second if it is better than the first

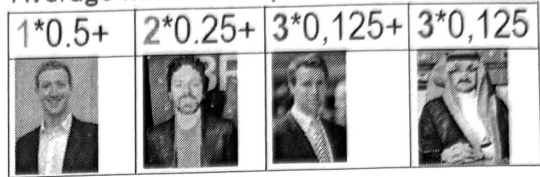


Say NO to the third only if it is worse than all the others

The rules at the top describe a secretary problem. Goal is to choose the best option from the candidates. Strategy is you must reject a baseline number of initial candidates. After that, then see who is better. Average number of questions =



Average number of questions =  $2 \times 0.25 + 2 \times 0.25 + 2 \times 0.25 + 2 \times 0.25 = 2$

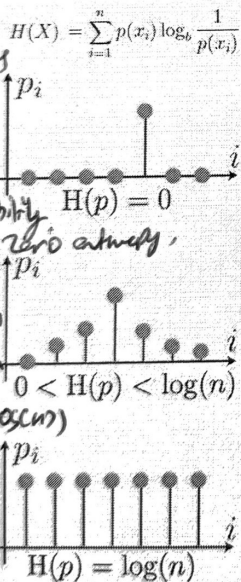


Question 1. Is this Zuckerberg?	50%	$1 \times 0.5$
Question 2. Is this Sergey Brin?	25%	$2 \times 0.25$
Question 3. Is this Stefan from BMW?	12.5%	$3 \times 0.125$
So Prince Saud	12.5%	$3 \times 0.125$
Average number of questions = 1.75		

Here, probabilities are shown and skewed so 25, 12.5, 12.5. Asking targeted questions about the most likely candidates saves the average to 1.75 questions. Huffman coding is used

Demonstration on how to optimize a secretary in many decision trees, if all 4 choices have an equal chance 25%, then it takes 2.49 on average to find an answer.

This visualizes how entropy changes based on probability



$$\sum_{i=1}^n p(i) \log_2 \frac{1}{p(i)}$$

Quantifying information

$$I(x_i) = \log_2 \left( \frac{1}{p_i} \right)$$

number of bits required to encode choice

$$\sum_{i=1}^n p(x_i) I(x_i)$$



Claude Shannon, defines information content  $I(x_i)$  (number of bits needed to encode choice) and Entropy  $H(x)$ . Average gainability of entropy.

50 billion	25 billion	12.5 billion	12.5 billion
Mark Zuckerberg	Sergey Brin	Stefan Quandt	Prince Al Saud
$P(1) = 50\%$	$P(2) = 25\%$	$P(3) = 12.5\%$	$P(4) = 12.5\%$

♠♠♠♠ =  $\frac{4}{8}$

♥ =  $\frac{1}{2}$  Entropy?

++ =  $\frac{2}{8} \cdot \log_2(4) = 0.25 \times 2$

◇ =  $\frac{1}{8} \cdot \log_2(8) = 0.375$

$$0.5 \times \log_2(2) + 0.25 \times \log_2(4) + 0.125 \times \log_2(8) + 0.125 \times \log_2(8) = 0.5 \times 1 + 0.25 \times 2 + 0.125 \times 3 + 0.125 \times 3 = 0.5 + 0.5 + 0.375 + 0.375 = 1.75$$